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## 1. The Theorem of Phythagoras

## Preparations

- Open a new GeoGebra file.
- Hide the algebra window, input field, and coordinate axes (View menu).
- Change the labeling setting to New points only (menu Options - Labeling).



## Instructions

| 1 | Segment a with endpoints $A B$ |
| :---: | :---: |
| 2 | Semicircle $c$ through points $A$ and $B$ |
| 3 | New point $C$ on the semicircle <br> Hint: Check if point $C$ really lies on the arc by dragging it with the mouse. |
| 4 | Hide the segment and the semicircle. |
| 5 | Triangle $A B C$ in counterclockwise direction |
| 6 | Rename the triangle sides to $a, b$, and $c$. |
| 7 | Interior angles of triangle $A B C$ <br> Hint: Click in the middle of the polygon to create all angles. |
| 8 | Drag point $C$ to check if your construction is correct. |
| 9 | Perpendicular line d to segment $B C$ through point $C$ |
| 10 | Perpendicular line e to segment $B C$ through point $B$ |
| 11 | Circle f with center $C$ through point $B$ |
| 12 | Intersect circle $f$ and perpendicular line $d$ to get intersection point D |
| 13 | Parallel line g to segment $B C$ through point $D$ |
| 14 | Intersection point $E$ of lines $e$ and $g$ |


| 15 |  | Square CBED |
| :--- | :--- | :--- |
| 16 | O | Hide the auxiliary lines and circle. |
| 17 |  | Repeat steps 8 to 15 for side $A C$ of the triangle. |
| 18 |  | Repeat steps 8 to 15 for side $A B$ of the triangle. |
| 19 | Drag the vertices of the right triangle to check if your squares are <br> correct. |  |
| 20 | Enhance your construction using the Properties dialog. |  |

## Enhancing the construction

Insert static and dynamic text into your construction that helps to understand the Pythagorean theorem $a^{2}+b^{2}=c^{2}$ where $a$ and $b$ are the legs and $c$ is the hypothenuse of a right triangle.


## Introduction of a new tool

Hint: Click on an object to copy its visual style. Then, click on other objects to match their visual style with the first object.

Hints: Don't forget to read the toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

## Instructions

| 21 | $\bullet \bullet$ | Create the midpoints of all three squares <br> Hint: Click on diagonal opposite vertices of each square. |
| :--- | :--- | :--- | :--- |
| 22 | ABC | Insert static text1: "a^2" and attach it to the midpoint of the <br> corresponding square <br> Hint: Don't forget to check the box LaTeX formula to get $a^{2}$. |
| 23 | ABC | Insert static text2: "b^2" and attach it to the midpoint of the <br> corresponding square |
| 24 | ABC | Insert static text1: "C^2" and attach it to the midpoint of the <br> corresponding square |
| 25 | O | Hide the midpoints of the squares. |


| 26 | Format the text to match the color of the corresponding squares. |  |
| :--- | :--- | :--- |
| 27 | ABC | Insert text that describes the Pythagorean theorem. |
| 28 | Export your construction as a dynamic worksheet. Come up with an <br> explanation that helps your students understand the theorem of <br> Pythagoras. |  |

## 2. Creating Custom Tools

GeoGebra allows you to create custom tools. This means that you can extend the toolbar by creating your own tools. Let's now create a tool that determines the minimal distance between a line and a point (e.g. altitude in a triangle). Before you can create your custom tool you need to construct all the objects required for your tool.

## Prepare the construction

## Preparations

- Open a new GeoGebra file.
- Hide the algebra window, input field, and coordinate axes (View menu).
- Change the labeling setting to All new objects (menu Options - Labeling).


## Instructions

| 1 |  | Line a through two points $A$ and $B$ |
| :--- | :--- | :--- |
| 2 | A | New point $C$ |
| 3 |  | Perpendicular line $b$ to line $a$ through point $C$ |
| 4 |  | Intersection point $D$ of lines $a$ and $b$ |
| 5 |  | Segment $c$ between points $C$ and $D$ |
| 6 | Drag points $A, B$, and $C$ to check your construction. |  |
| 7 |  | Change the color of segment $c$ and hide the labels of all objects. |

## Create a custom tool

1. In menu Tools click on Create new tool... to open the Create new tool dialog window.
2. By default, tab Output Objects is activated.
3. Specify the output objects of your new tool by either clicking on the desired output object on the drawing pad (our example: segment $c$ ) or selecting it from the drop down menu (click on the little arrow next to the input field).
4. Click the Next > button in order to activate tab Input Objects.
5. GeoGebra fills in the corresponding input objects for your tool automatically (our example: points $A, B$, and $C$ ).
Note: GeoGebra picks all so-called 'parent objects' of the output objects you specified.

6. Click the Next > button in order to activate tab Name \& Icon.
7. Fill in a name for your tool and text for the toolbar help.
Note: GeoGebra fills in the text field Command name automatically.
8. Click the button Finish.

Note: Your new tool is now part of the GeoGebra toolbar.


## Try out your custom tool

1. Open a new GeoGeba file using menu File - New. Note: Your custom tool is still part of the tool bar.
2. Create a triangle $A B C$ using tool Polygon.
3. Activate your custom tool Altitude.
4. Click on points $A, B$, and $C$ in order to create one of the triangle's altitudes.
5. Create another altitude of the triangle.
6. Intersect the two altitudes to get the orthocenter of the triangle.

## 3. Saving and Importing Custom Tools

## Save your custom tool

1. In menu Tools click on Manage tools... to open the Manage tools dialog window.
2. Select your custom tool Altitude from the list of available tools.
3. Click on button Save as... in order to save your custom tool and make it available for future constructions.
4. Choose a name for your custom tool (e.g. Altitude_tool.ggt) and save it on your computer.

Note: Custom GeoGebra tools are saved with the file name extension .ggt. This helps you to distinguish between 'usual' GeoGebra files (extension .ggb) and custom tool files.

## Import a custom tool

After saving your custom tool you are able to reuse it in future constructions. By default the GeoGebra tool bar doesn't include any custom tools. In order to reuse one of your custom tools you need to import it into your new GeoGebra file.

1. Open a new GeoGebra window (menu File $-\square$ New window).
2. In menu File click on Open.
3. Look for the custom tool you saved earlier (e.g. Altitude_tool.ggt) and select it from the list of available GeoGebra files (.ggb) and tool files (.ggt).
4. Click the Open button to import your custom tool into the toolbar of the new GeoGebra window.

Note: Importing a custom tool doesn't affect the construction in your GeoGebra window. Thus, you can also import custom tools during a construction process.

## 4. Creating a Square Tool

## Preparations

- Open a new GeoGebra file.
- Hide the algebra window, input field, and coordinate axes (View menu).
- Change the labeling setting to All new objects (menu Options - Labeling).


## Instructions

| 1 |  | Segment a with endpoints $A B$ |
| :--- | :--- | :--- |
| 2 |  | Perpendicular line $b$ to segment $A B$ through point $B$ |

## Task

Compare the construction process of this square with the one you used in workshop 2. What are the differences?

## 5. The Fibonacci Spiral

A Fibonacci spiral can be created by drawing arcs connecting the opposite corners of squares in the Fibonacci tiling which uses squares of sizes $1,1,2,3,5$, $8,13,21, \ldots$


The Fibonacci spiral approximates the so called Golden Spiral which is a logarithmic spiral whose growth factor is related to the golden ratio.

## Preparations

- Open a new GeoGebra window.
- Import your Square tool into the toolbar (menu File - Open).
- Hide the algebra window, input field, and coordinate axes (View menu).
- Show the grid (View menu).
- Change the labeling setting to No new objects (menu Options - Labeling).


## Introduction of a new tool

| Circular arc with center through two points $\quad$ New! |  |
| :--- | :--- |
| - | Hint: Click on the center point of the circular arc. Then, specify two points that <br> determine the radius and length of the arc. |

Hints: Don't forget to read the toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

## Instructions

| 1 | O | Use your Square tool to create a square with side length 1 <br> Hint: Place the two points on grid points that are next to each other. |
| :---: | :---: | :---: |
| 2 | $\sigma$ | Create a second square with side length 1 below the first square. <br> Hint: Use already existing points to connect both squares. |
| 3 | 0 | Create a third square with side length 2 on the right hand side of the two smaller squares. |
| 4 | $\mathscr{S}_{6}$ | Continue creating squares with side lengths $3,5,8$, and 13 in counter clockwise direction. |
| 5 |  | Create a circular arc within the first square you created. <br> Hint: Specify the lower right vertex of the square as the center of the arc. Select two opposite vertices of the square in counter clockwise orientation. |
| 6 |  | Repeat step 5 for each of the squares in order to construct the Fibonacci spiral. |
| 7 |  | Enhance your construction using the Properties dialog. |

## 6. Constructing the Center of a Circle

## Back to school...

Do you know how to construct the center of a circle? Use the circles provided by your workshop presenter (4 per participant) and try to find a way of finding the center of these circles (a) only by folding the paper and (b) with pencil and ruler.

## Hints:

- Version 1a: Fold two circle diameters which intersect in the circle's center.
- Version 1b: Can you recreate this construction using pencil and ruler?
- Version 2a: Fold two chords of the circle as well as their perpendicular bisectors which intersect in the center of the circle.
- Version 2b: Can you recreate this construction using pencil and ruler?

Now use GeoGebra in order to recreate the construction you used in (2b).

## Preparations

- Open a new GeoGebra file.
- Hide the coordinate axes and the algebra window, but show the input field (View menu).


## Instructions

| 1 | Enter circle c's equation: $\mathrm{x}^{\wedge} 2+\mathrm{y}^{\wedge} 2=16$ |
| :---: | :---: |
| 2 | Create chord $a$ of circle $c$ <br> Hint: A chord is a segment whose endpoints both lie on the circle. |
| 3 | Create midpoint $C$ of chord $a$. |
| 4 | Create perpendicular line $b$ to the chord $a$ through point $C$. <br> Hint: You just created the perpendicular bisector of chord a. |
| 5 | Create another chord $d$ of circle $c$. |
| 6 | Create midpoint $F$ of chord $d$. |
| 7 | Create perpendicular line $e$ to chord $d$ through point $F$. |


| 8 |  | Intersect lines $b$ and $e$ to get intersection point G. |
| :--- | :--- | :--- |
| 9 |  | Enhance your construction using the Properties dialog. |
| 10 | Check your construction for different positions of the chords. |  |

## Tasks

- Show the Navigation bar within the graphics window of your construction and use it to review the construction steps.
- Insert breakpoints in the Construction protocol to group some of the objects you used (open the Construction protocol View menu - Breakpoint). After specifying your breakpoints, check
 Show only breakpoints in the View menu of the Construction protocol.
- Export the construction as a dynamic worksheet that includes the Navigation bar (Export dialog - tab Advanced) and save the file as Center-Circle-Solution.html.
- Open the dynamic worksheet you just exported. Use the Navigation bar to review your construction and write down which tools you used in order to construct the center of the circle.


## 7. Customizing the Toolbar

You will now learn how to customize GeoGebra's toolbar so you can limit the number of available tools for your students.


1. In the Tools menu click on Customize toolbar...
2. The window on the left hand side lists all GeoGebra tools that are part of the default toolbar. If you click on one of the + symbols in front of the tool names the corresponding toolbox is opened. The window on the right hand side contains all available GeoGebra tools.
3. In the left hand side list select the word Separator. Click button Remove> several times until all list entries apart from Move are deleted.
4. In the left hand side list click on the + symbol in front of the Move tool to open the toolbox. Select tool Rotate around point and click the Remove> button again. The Move tool should now be the only tool left in the list on the left hand side.
5. Close the Move toolbox in the left hand side list by clicking on the symbol.
6. In the right hand side list select tool $X$ Intersect two objects and click button Insert <
7. Repeat step 6 for the other tools you need for constructing a circle's center $\left(\bullet^{\circ}\right.$ Midpoint or center, ${ }^{-}$Segment between two points, $\dagger$ Perpendicular line).
8. Use the Up and Down buttons to change the order of the tools in the left hand side list.
9. Click Apply once you are done.
10. Your GeoGebra window should now show the customized toolbar.


## Task

- Delete all objects apart from the circle.
- Export this updated construction as a dynamic worksheet that includes the customized toolbar and shows the toolbar help (Export dialog - tab Advanced).
- Save the dynamic worksheet as Center-Circle-Construction.html.


## 8. Challenge of the Day: Euler's Discovery

## Task

- Construct the following three 'remarkable' points of a triangle: circumcenter, orthocenter, and centroid. Create a custom tool for each of these points. Save your custom tools.
- Use your custom tools within one construction to find the relation between these three points as the Swiss mathematician Euler did in the $18^{\text {th }}$ century (obviously without having access to dynamic geometry software ;-)


## Circumcenter of a Triangle

## Preparations

- Open new GeoGebra file.
- Hide algebra window, input field and coordinate axes (View menu).
- Change the labeling setting to New points only (menu Options Labeling).



## Instructions

| 1 | Arbitrary triangle $A B C$ |
| :---: | :---: |
| 2 | Line bisectors $d, e$, and $f$ for all sides of the triangle <br> Hint: The tool Line bisector can be applied to an existing segment. |
| 3 | Intersection point $D$ of the two of the line bisectors. <br> Hint: The tool, Intersect two objects, can't be applied to the intersection of three lines: either select two of the three line bisectors successively, or click on the intersection point and select one line at a time from the appearing list of objects in this position. |
| 4 | Circle with center $D$ through one of the vertices of triangle $A B C$ |
| 5 | Rename point $D$ to Circumcenter. |
| 6 | Use the drag test to check if your construction is correct |
| 7 | Create a custom tool for the circumcenter of a triangle. |


|  | Output objects: point Circumcenter <br>  <br>  <br> 8Input objects: points $A, B$, and $C$ <br> Name: Circumcenter <br> Toolbar help: Click on three points |
| :--- | :--- |
| Save your custom tool as file circumcenter.ggt. |  |

## Orthocenter of a Triangle

## Preparations

- Open new GeoGebra file.
- Hide algebra window, input field and coordinate axes (View menu).
- Change the labeling setting to New points only (menu Options - Labeling).



## Instructions

| 1 | Arbitrary triangle $A B C$ |  |
| :--- | :--- | :--- |
| 2 |  | Perpendicular lines $d, e$ e, and $f$ to each side through the opposite <br> vertex of the triangle |
| 3 | Intersection point $D$ of two of the perpendicular lines |  |
| 4 | b | Rename point $D$ to Orthocenter. |
| 5 |  | Use the drag test to check if your construction is correct |
| 6 | S | Create a custom tool for the orthocenter of a triangle. <br> Output objects: point Orthocenter |
|  | Onput objects: points $A, B$, and $C$ |  |
| 7 | Iname: Orthocenter <br> Toolbar help: Click on three points |  |

## Centroid of a Triangle

## Preparations

- Open new GeoGebra file.
- Hide algebra window, input field and coordinate axes (View menu).

- Change the labeling setting to New points only (menu Options - Labeling).


## Instructions

| 1 | Arbitrary triangle $A B C$ |
| :---: | :---: |
| 2 | Midpoints $D, E$, and $F$ of the triangle sides |
| 3 | Connect each midpoint with the opposite vertex using segments $d, e$, and $f$ |
| 4 | Intersection point $G$ of two of two of the segments |
| 5 | Rename point G to Centroid. |
| 6 | Use the drag test to check if your construction is correct |
| 7 | Create a custom tool for the centroid of a triangle. <br> Output objects: point Centroid <br> Input objects: points $A, B$, and $C$ <br> Name: Centroid <br> Toolbar help: Click on three points |
| 8 | Save your custom tool as file centroid.ggt. |

## What was Euler's discovery?

## Task 1

- Open a new GeoGebra file and import your three custom tools (circumcenter.ggt, orthocenter.ggt, and centroid.ggt) into the toolbar.
- Create an arbitrary triangle ABC and apply all three custom tools to the triangle in order to create the circumcenter, orthocenter, and centroid within the same triangle.
- Move the vertices of triangle $A B C$ and observe the three 'remarkable' points you just constructed. Which relationship do they have? Use one of GeoGebra's geometry tools in order to visualize this relationship.


## Task 2

- Open an empty GeoGebra file. Customize the toolbar so it only consists of the following tools: Move, Polygon, Line through two points, Circle with center through point, Circumcenter, Orthocenter, and Centroid.
- Export this empty GeoGebra file as a dynamic worksheet that includes the customized toolbar as well as the toolbar help. Come up with instructions that guide your students towards discovering the Euler line in a triangle


## Constructing the Center of a Circle Worksheet



