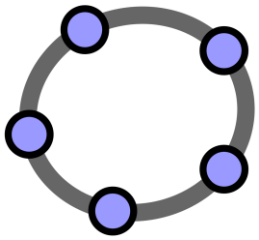

Geometric Constructions & Use of Commands

GeoGebra Workshop Handout 2

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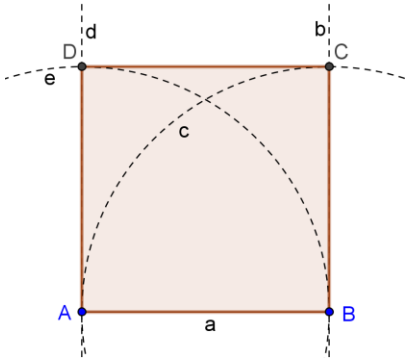
Table of Contents

1. Square Construction



Preparations

- Open a new GeoGebra file.
- Hide algebra window, input field and coordinate axes (*View* menu).
- Change the labeling setting to *New points only* (menu *Options* – *Labeling*).

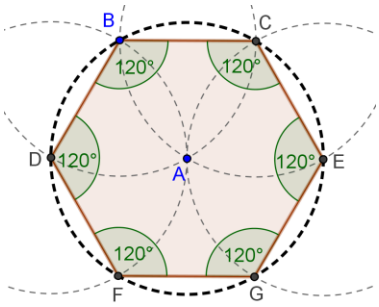
Instructions

1		Segment *AB* between points *A* and *B*
2		Perpendicular line *b* to segment *AB* through point *B*
3		Circle c with center *B* through point *A*
4		Intersect circle *c* with perpendicular line *b* to get intersection point *C*
5		Perpendicular line d to segment *AB* through point *A*
6		Circle e with center *A* through point *B*
7		Intersect perpendicular line *d* with circle *e* to get intersection point *D*
8		Create polygon *ABCD* Hint: Don't forget to close the polygon by clicking on point *A* after selecting point *D*.
9		Hide circles and perpendicular lines
10		Use the drag test to check if your construction is correct

Challenge: Can you come up with a different way of constructing a square?

2. Regular Hexagon Construction

Preparations



- Open a new GeoGebra file.
- Hide algebra window, input field and coordinate axes (*View* menu).
- Change the labeling setting to *All new objects* (menu *Options* – *Labeling*).

Instructions

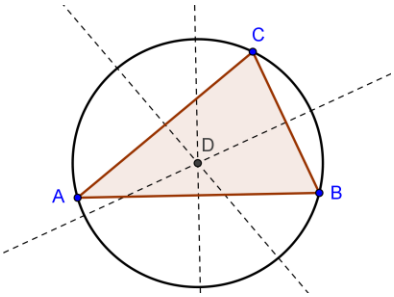
1		Circle c with center A through point B
2		Circle d with center B through point A
3		Intersect the circles c and d to get the hexagon's vertices C and D
4		Circle e with center C through point A
5		Intersect the new circle e circle c in order to get vertex E
6		Circle f with center E through point A
7		Intersect the new circle f with circle c in order to get vertex F
8		Circle g with center F through point A
9		Intersect the new circle g with circle c in order to get vertex G
10		Draw hexagon FGDBCE
11		Hide the circles
12		Display the interior angles of the hexagon
13		Use the drag test to check if your construction is correct

Task: Try to find an explanation for this construction process.
Hint: Which radius do the circles have and why?

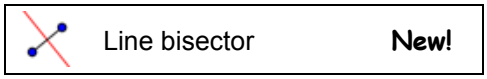
3. Circumscribed Circle of a Triangle

Preparations

- Open new GeoGebra file.
- Hide algebra window, input field and coordinate axes (*View* menu).
- Change the labeling setting to *New points only* (menu *Options* – *Labeling*).



Introduction of new tool



Hints: Don't forget to read the toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

Instructions

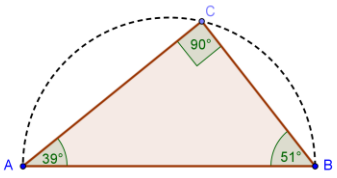
1		Arbitrary triangle *ABC*
2		Line bisector for each side of the triangle
3		Intersection point *D* of two of the line bisectors Hint: The tool, Intersect two objects, can't be applied to the intersection of three lines. Either select two of the three line bisectors successively, or click on the intersection point and select one line at a time from the appearing list of objects in this position.
4		Circle with center *D* through one of the vertices of triangle *ABC*
5		Use the drag test to check if your construction is correct

Back to school…

Modify your construction to answer the following questions:

1. Can the circumcenter of a triangle lie outside the triangle? If yes, for which types of triangles is this true?
2. Try to find an explanation for using line bisectors in order to create the circumcenter of a triangle.

4. Visualize the Theorem of Thales



Back to school…

Before you begin this construction, check out the dynamic worksheet called *02_Theorem_Thales.html* in order to see how students could rediscover what the Greek philosopher and mathematician Thales found out about 2600 years ago.

Preparations

- Open new GeoGebra file.
- Hide algebra window, input field and coordinate axes (*View* menu).
- Change the labeling setting to *New points only* (menu *Options – Labeling*).

Introduction of a new tool

	**Semicircle through two points** **New!** Hint: The order of clicking points A and B determines the direction of the semicircle.

Hints: Don't forget to read the toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

Instructions

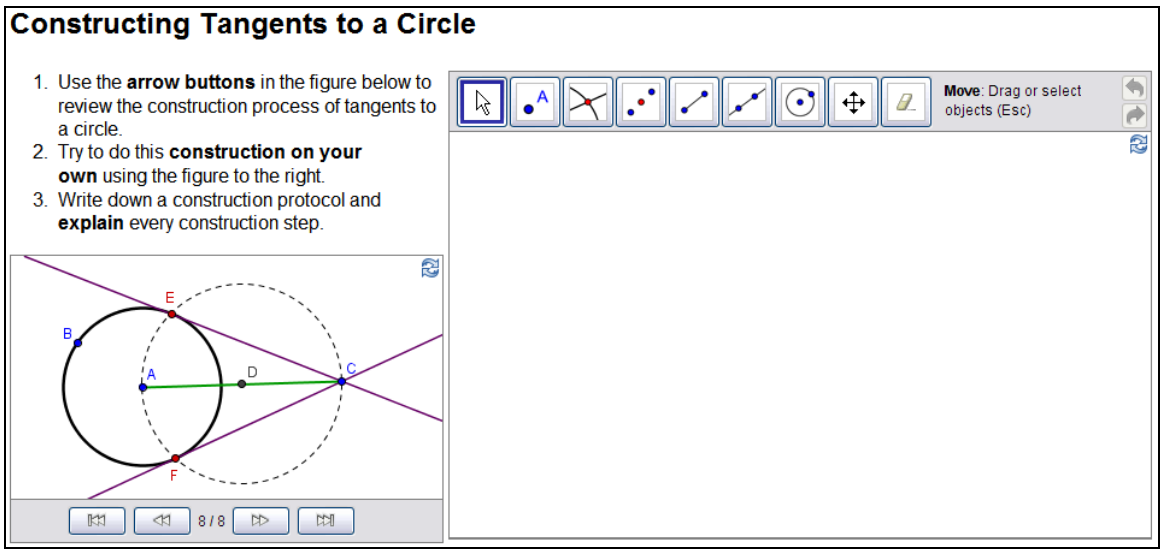
1		Segment *AB*
2		Semicircle through points *A* and *B*
3	 	New point *C* on the semicircle Hint: Check if point *C* really lies on the arc by dragging it with the mouse.
4		Create triangle *ABC* in counterclockwise direction
5		Create the interior angles of triangle *ABC* Hint: Click in the middle of the polygon.
6		Drag point *C* to check if your construction is correct.

Task: Try to come up with a graphical proof for this theorem.
Hint: Create midpoint *O* of segment *AB* and display the radius *OC* as a segment.

5. Constructing Tangents to a Circle

Back to school…

Open the dynamic worksheet *03_Tangents_Circle.html*. Follow the directions on the worksheet in order to find out how to construct tangents to a circle.



What if my mouse and touchpad wouldn't work?

Imagine your mouse and / or touchpad stop working while you are preparing GeoGebra files for tomorrow's lesson. How can you finish the construction file?

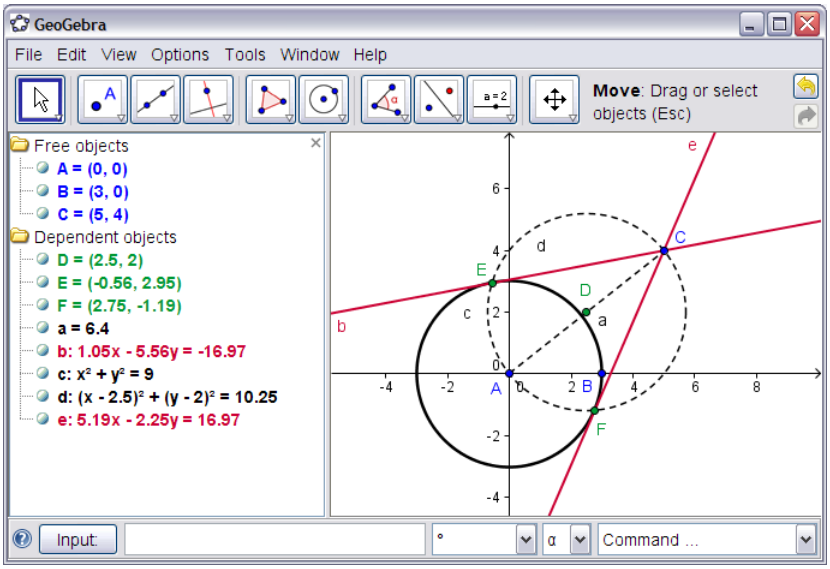
GeoGebra offers algebraic input and commands in addition to the geometry tools. Every tool has a matching command and therefore, could be applied without even using the mouse.
Note: GeoGebra offers more commands than geometry tools. Therefore, not every command has a corresponding geometry tool!

Task: Check out the list of commands next to the input field and look for commands whose corresponding tools were already introduced in this workshop.

As you saw in the last activity, the construction of tangents to a circle can be done by using geometric construction tools only. You will now recreate this construction by just using keyboard input.

Preparations

- Open a new GeoGebra file.
- Show the algebra window and input field, as well as coordinate axes (*View* menu)



Instructions

1	`A = (0, 0)`	Create point *A* Hint: Make sure to close the parenthesis.
2	`(3, 0)`	Create point *B* Hint: If you don't specify a name objects are named in alphabetical order.
3	`c = Circle[A, B]`	Circle with center *A* through point *B* Hint: Circle is a dependent object

Note: GeoGebra distinguishes between free and dependant objects. While free objects can be directly modified either using the mouse or the keyboard, dependant objects adapt to changes of their parent objects. Thereby, it is irrelevant in which way (mouse or keyboard) an object was initially created!

Task 1: Activate *Move* mode and double click an object in the algebra window in order to change its algebraic representation using the keyboard. Hit the *Enter* key once you are done.

Task 2: You can use the arrow keys in order to move free objects in a more controlled way. Activate *Move* mode and select the object (e.g. a free point) in either window. Press the up / down or left / right arrow keys in order to move the object into the desired direction.

4	`C = (5, 4)`	Point *C*
5	`s = Segment[A, C]`	Segment *AC*
7	`D = Midpoint[s]`	Midpoint *D* of segment *AC*
8	`d = Circle[D, C]`	Circle with center *D* through point *C*
9	`Intersect[c, d]`	Intersection points *E* and *F* of the two circles
10	`Line[C, E]`	Tangent through points *C* and *E*
11	`Line[C, F]`	Tangent through points *C* and *F*

Checking and enhancing the construction

- Perform the drag-test in order to check if the construction is correct.
- Change properties of objects in order to improve the construction's appearance (e.g. colors, line thickness, auxiliary objects dashed,…).
- Save the construction.

6. Exploring Parameters of a Quadratic Polynomial

Back to school…

In this activity you will explore the impact of parameters on a quadratic polynomial. You will experience how GeoGebra could be integrated into a 'traditional' teaching environment and used for active, student-centered learning.

Follow the instructions on the paper worksheet and write down your results and observations while working with GeoGebra. Your notes will help you during the following discussion of this activity.

Exploring Parameters of a Quadratic Polynomial

1. Open a **new GeoGebra file**.

2. **Type** in `f(x) = x^2` and hit the *Enter* key. Which **shape** does the function graph have? Write down your answer on paper.

3. In *Move* mode, highlight the polynomial in the algebra window and use the ↑ **up and** ↓ **down arrow keys**.
   a. How does this **impact the graph** of the polynomial? Write down your observations.
   b. How does this **impact the equation** of the polynomial? Write down your observations.

4. Again, in *Move* mode, highlight the function in the algebra window and use the ← **left and** → **right arrow keys**.
   a. How does this **impact the graph** of the polynomial? Write down your observations.
   b. How does this **impact the equation** of the polynomial? Write down your observations.

5. In *Move* mode, double click the equation of the polynomial. Use the keyboard to **change the equation** to `f(x) = 3 x^2`.
   Hint: Use an asterisk * or space in order to enter a multiplication.
   a. **Describe** how the function graph changes.
   b. **Repeat changing the equation** by typing in different values for the parameter (e.g. 0.5, -2, -0.8, 3). **Write down** your observations

7. Using Sliders to Modify Parameters

Let's try out a more dynamic way of exploring the impact of a parameter on a polynomial `f(x) = a x^2` by using sliders to modify the parameter values.

Preparations

- Open a new GeoGebra file.
- Show the algebra window, input field, and coordinate axes (*View* menu).

Instructions

1		Create the variable `a = 1`
2		Display the variable *a* as a slider in the graphics window Hint: you need to right click (MacOS: *Ctrl*-click) the variable in the algebra window and select *Show object*.
3		Enter the quadratic polynomial `f(x) = a * x^2` Hint: Don't forget to enter an asterisk * or space between `a` and `x^2`.
4		Create *a* slider *b* using the *Slider* tool Hint: Activate the tool and click on the drawing pad. Use the default settings and click *Apply*.
5		Enter the polynomial `f(x) = a * x^2 + b` Hint: GeoGebra will overwrite the old function *f* with the new definition.

8. Challenge of the Day: Parameters of Polynomials

Use the file created in the last activity in order to work on the following tasks:

- Change the parameter value *a* by moving the point on the slider with the mouse. How does this influence the graph of the polynomial? What happens to the graph when the parameter value is
  - (a) greater than 1,
  - (b) between 0 and 1, or
  - (c) negative?

  Write down your observations.
- Change the parameter value *b*. How does this influence the graph of the polynomial?

- Create a slider for a new parameter *c*. Enter the quadratic polynomial **`f(x) = a * x^2 + b x + c.`** Change the parameter value c and find out how this influences the graph of the polynomial.

9. Tips and Tricks

Basic use of GeoGebra

- Summarize the properties of the geometric figure you want to create.
- Try to find out which GeoGebra tools can be used in order to construct the figure using some of these properties (e.g. right angle – tool Perpendicular line).
- Make sure, you know how to use each tool before you begin the construction. If you don't know how to operate a certain tool, activate it and read the toolbar help.
- For each of these activities, open a new GeoGebra file, hide the algebra window, input field, and the coordinate axes.
- You might want to save your files before you start a new activity.
- Don't forget about the Undo and Redo buttons in case you make a mistake.
- Frequently use the Move tool in order to check your construction (e.g. are objects really connected, did you create any unnecessary objects).
- If you have questions, please ask a colleague before you address the presenter or assistant(s).

Algebraic input and commands

- **Name a new object** by typing in `name =` in front of its algebraic representation. Example: `P = (3, 2)` creates point *P*.
- **Multiplication** needs to be entered using an asterisk or space between the factors. Example: `a*x` or `a x`
- **GeoGebra is case sensitive**! Thus, upper and lower case letters must not be mixed up. Note:
  - Points are always named with upper case letters
    Example: `A = (1, 2)`
  - Segments, lines, circles, functions… are always named with lower case letters.
    Example: circle `c: (x - 2)^2 + (y - 1)^2 = 16`
  - The variable *x* within a function and the variables *x* and *y* in the equation of a conic section always need to be lower case.
    Example: `f(x) = 3*x + 2`

- If you want to use an **object within an algebraic expression** or command you need to create the object prior to using its name in the input field. Examples:
  - `y = m x + b` creates a line whose parameters are already existing values m and b (e.g. numbers / sliders).
  - `Line[A, B]` creates a line through existing points *A* and *B*.
- **Confirm an expression** you entered into the input field by pressing the *Enter* key.
- **Open the help window** for using the input field and commands by clicking the question mark to the left of the input field.
- **Error messages**: Always read the messages – they could possibly help to fix the problem!
- **Commands** can be typed in or selected from the list next to the input field. Hint: If you don't know which parameters are required within the brackets of a certain command, type in the full command name and press key *F1*. A pop-up window appears explaining the syntax and necessary parameters of the command.
- **Automatic completion of commands**: After typing in the first two letters of a command into the input field, GeoGebra tries to complete the command.
  - If GeoGebra suggests the desired command, hit the *Enter* key in order to place the cursor within the brackets.
  - If the suggested command is not the one you wanted to enter, just keep typing until the suggestion matches.